

Comments on “Resource placement in
Cartesian product of networks” [Imani,
Sarbaz-Azad and Zomaya, J. Parallel Distrib.
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Abstract: The present note points out a number of errors, omissions, redundancies and arbitrary deviations from the standard terminology in the paper “Resource placement in Cartesian product of networks,” by N. Imani, H. Sarbaz-Azad and A.Y. Zomaya [J. Parallel Distrib. Comput. 70 (2010) 481-495].

Key words: Critique, comments, resource placement, Cartesian product, graphs and networks.

1 Introduction

The present note is a critique of the paper “Resource placement in Cartesian product of networks” by Imani, Sarbazi-Azad and Zomaya [3]. The paper under review contains a number of errors, omissions, redundancies and arbitrary deviations from the standard terminology. In particular, the authors (a) present certain results that have existed in the literature for ages, (b) do not cite relevant references, (c) indulge in unnecessary redundancies, and (d) make statements that are vague, meaningless or incorrect. In addition, there are a number of grammatical errors (not covered in the present study).

In what follows, there is a section-by-section commentary on the paper under review.

2 Comments on Section 2

1. In the fourth paragraph from the bottom (left column, p. 482), the authors refer to a paper by Alrabady et al [1] for certain definitions of resource allocation strategies. Interestingly enough, the forenamed paper itself contains several errors. For example,
 - (a) Lemma 1 [1] (p. 62): “Any perfect resource set is a dominating set and a maximal independent set.” The plain truth is that a maximal independent set is necessarily a dominating set.
 - (b) At the top of the second column [1] (p. 62): “... looking for a dominating set and a maximal independent set is an NP-complete problem,” which is far from true. Indeed, obtaining a smallest dominating set, or a smallest/largest maximal independent set is NP-complete.
2. In the third paragraph from the bottom (left column, p. 482), replace “less than or equal to d ” by “greater than d .”
3. On the sixth line from the bottom (left column, p. 482), replace “ $d = 1$ and $m = 1$ ” by “ $m = 1$ and $d = 1$.”
4. Definition 2 (right column, p. 482) deals with $Vol_G(d, c)$ that is the number of vertices within a distance of d from a fixed vertex c in G . In general, $Vol_G(d, c_1)$ need not be equal to $Vol_G(d, c_2)$ for $c_1 \neq c_2$,

yet the authors make use of $Vol_G(d, c)$ in the statement and proof of Theorem 1 without ever referring to vertex c . They implicitly assume that $Vol_G(d, c)$ is independent of c , which need not be true with respect to an arbitrary graph G . Note that the statement of Theorem 1 starts with “For any graph $G = (V, E) \dots$ ”

5. The foregoing concept of *volume* is not used anywhere in Sections 4 and 5, which constitute the main body of the paper, so the definition itself is useless.
6. On p. 482 (second column), the authors introduce the term “homogeneous” in the sense of “isomorphic.” When there exists a world-wide unanimity on the concept of isomorphism, introducing a different term for that purpose is patently incorrect. Strangely enough, the authors themselves, in an earlier paper [4], employed the terms “isomorphic” and “isomorphism” in the usual sense. Moreover, there already exists the concept of a homogeneous graph in the literature that is completely different from that of isomorphism: *A graph G is said to be homogeneous if for any two isomorphic vertex-induced subgraphs $\langle X \rangle$ and $\langle Y \rangle$ of G , there exists some isomorphism between $\langle X \rangle$ and $\langle Y \rangle$ that extends to an automorphism of G [2].*
7. Corollary 1 (p. 482) deals with the vertex partition of a Cartesian product of several graphs into subgraphs isomorphic to a fixed factor graph. A detailed proof without any citation purports that this is authors’ original idea. However, this property (and the related concept of projection on a fixed co-ordinate) of a Cartesian-product graph has existed in the literature for a long time [7, 8]. In particular, it was illustrated in the book on product graphs by Imrich and Klavžar [5] (pp. 30-31), which was published more than ten years before the publication of the paper under review, yet the authors do not cite that book or any other source for that purpose. Interestingly enough, they cite a paper jointly by Klavžar [6] that itself refers to that book.

3 Comments on Section 3

1. Definition 3 (p. 483): Replace “ $0 \leq i \leq |Q_G|$ ” by “ $1 \leq i \leq |Q_G|$ ”.
2. Proof of Corollary 2 (p. 483):

- (a) Replace $Q_G(k, d)$ by $Q_G(m, d)$.
 - (b) Replace “ R contains” by “each R_i contains”.
3. The proof of Corollary 3 (p. 483) is completely redundant since the statement is an obvious consequence of Definition 3 and Corollary 2. Likewise Corollary 4 (p. 483) follows from the fact that Q_G constitutes a vertex partition of G , so its proof is equally redundant.

4 Comments on Section 4

1. On p. 484, the authors present Algorithm 2-*HMP*, which consists of steps (a) and (b). Immediately thereafter, they prove in Theorem 2 that Step (b) of that algorithm is redundant. An identical situation arises on p. 485 with respect to Algorithm 2-*HTP* and Theorem 3, respectively. This kind of baroque has no place in a journal where space is at a premium. The authors have a responsibility to present an algorithm succinctly, so there is no chaff around it.
2. In the proof of Theorem 4 (p. 485), the authors write, “ \dots a bijective function, i.e., a function that is both surjective and injective.” There is absolutely no need to educate the reader of a premium journal that a bijective function is both surjective and injective.
3. In the proof of Theorem 4 (p. 485), the authors write, “ \dots the output of our algorithm is a $|Q_H| = |Q_{G_1}| = |Q_{G_2}|$ cubic matrix M ,” which does not make sense at all. Indeed, there is no “cubic matrix” anywhere else in the paper. A little later, they present Algorithm *DM-MF* (that is a part of the proof of the same theorem) in which M appears as the matrix $M_{|Q_H| \times |Q_H|}$. Further, at Step (5) of the same algorithm (p. 486), M appears as the matrix $M_{a \times b}$ without any subsequent discussion on how a and b are related to $|Q_H|$.
4. In the statements of Theorems 5 and 6 (pp. 486-487), the authors start with the hypothesis that G_1 and G_2 are arbitrary graphs, and then immediately impose the condition that $|Q_{G_1}| = |Q_{G_2}|$ where, in addition, there exists a bijection ϕ_1 from Q_{G_1} to Q_{G_2} . The condition is severe, hence at odds with the premise that G_1 and G_2 are arbitrary graphs.

5. In the first paragraph of the proof of Theorem 5 (p. 486), $v \in R_{1,1}$ as well as $v \in R_{H,1}$, where $R_{1,1} \in Q_{G_1}$ and $R_{H,1} \in Q_{G_1 \times G_2}$. It is impossible to reconcile the membership of v in both $R_{1,1}$ and $R_{H,1}$.
6. p. 487, second column: Replace “ $Q_{G_i} = Q_{G_j}$, $1 \leq i, j \leq k$, $i \neq j$ ” by “ $|Q_{G_i}| = |Q_{G_j}|$, $1 \leq i, j \leq k$ ”.
7. Step (b) of Algorithm *IHTP*(H) on p. 488 is as redundant as the respective step in each of Algorithm 2-*HMP* (p. 484) and Algorithm 2-*HTP* (p. 485).
8. In the proof of Theorem 9 (p. 488), “ $\bigcup_{i=1}^k V(G'_i) = u$ ” must be replaced by “ $\bigcap_{i=1}^k V(G'_i) = \{u\}$ ”.
9. Proof of Corollary 5 (p. 488) is trivial, hence unnecessary.

5 Comments on Section 5

1. On the fourth line in the paragraph after Algorithm *SDP* (p. 489): Replace “an arbitrary graph in H ” by “an arbitrary node in H ”.
2. In the paragraph immediately above Corollary 6 (p. 489): Replace “A known 2 distance-2 placement for C_6 is assumed” by “A known one-perfect distance-one placement for C_6 is assumed”.

References

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